DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS BY THE METHOD OF A MOMENTARY THERMAL PULSE AND TAKING INTO ACCOUNT THE THERMAL RESISTANCE IN CONTACTS

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A pulse method is described by which thermophysical characteristics of solid materials can be determined in the case of an imperfect thermal contact between layers of the test specimen and the reference specimen.

The main source of errors in the determination of the thermophysical characteristics of materials by comparative methods involving contacts are thermal resistances in the interface region along the boundary between the tested material and the reference standard. In order to decrease these errors, one usually endeavors to minimize the effect of contact resistances by careful treatment of the adjoining surfaces, by increasing the pressure in the contact region, by introduction of a high-conductivity material into the interface region, etc. [1-3]. However, all these measures do not always yield the desired results. This has been confirmed by estimates of the effect of thermal resistances in contacts on the experimental data obtained with thin specimens or with high-conductivity materials [4-5]. There was a method proposed [6] by which the thermal resistances in contacts could be eliminated in the determination of the thermophysical characteristics of high-conductivity materials. That method is an extension of the method of two temperature—time intervals [7], where a determination of the thermophysical characteristics is based on two arbitrary points on the temperature curve.

In this study will be considered the method of determining the thermophysical properties from the solution to the problem of propagation of a momentary thermal pulse through an infinitely large triple-layer medium with imperfect contacts between layers. The simplicity of realizing the boundary conditions makes it feasible to perform large-scale testing of specimens during a relatively short time. It is possible to calculate the thermophysical properties on the basis of an arbitrary number of points on the obtained temperature curve, which in turn makes it possible to improve the accuracy of the method by processing the experimental data in an only slightly more complex manner.

We consider the following problem. Within a layer of space ($0 < x < b$) there exists a momentary heat source with a power density

$$P(t) = Q\delta(t)/b \ [\mathrm{W/m^3}],$$

where Q is the energy emitted by the heater over a unit area of its surface; h, thickness of the layer which contains this heater; and $\delta(t)$, unit-impulse delta function.

The layer ($d < x < d + b$) contains a flat resistance thermometer of thickness b. The specific heat of the heater and of the thermometer, being much lower than that of the material specimen, will be disregarded. The thermal conductivity of the layers containing the heater and the thermometer, respectively, will be denoted as λ_r. The half-spaces ($-\infty < x < 0$) and ($d + b < x < \infty$) contain the reference material, whose thermal conductivity and thermal diffusivity are λ_0 and a_0, respectively. The tested material with the unknown properties λ and a is placed in the layer ($b < x < d$) between the heater and the thermometer. We will also assume that $b \ll d$. The functions of the temperature in these layers will be denoted as

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$$u(x,\ t)\ \text{at}\quad -\infty < x < 0,$$
$$v(x,\ t)\ \text{at}\quad b < x < d,$$
$$w(x,\ t)\ \text{at}\quad d+b < x < \infty.$$

The problem reduces to solving the system of heat-conduction equations

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{a_0}\frac{\partial u}{\partial t} = 0,$$
$$\frac{\partial^2 v}{\partial x^2} - \frac{1}{a}\frac{\partial v}{\partial t} = 0, \qquad (1)$$
$$\frac{\partial^2 w}{\partial x^2} - \frac{1}{a_0}\frac{\partial w}{\partial t} = 0$$

with the initial conditions

$$u(x,\ 0) = 0,\quad v(x,\ 0) = 0,\quad w(x,\ 0) = 0, \qquad (2)$$

the boundary conditions

$$\lambda_0 \frac{\partial u(0,\ t)}{\partial x} = \frac{\lambda_r}{b}[v(0,\ t) - u(0,\ t)] + \frac{Q}{2}\delta(t),$$
$$\lambda \frac{\partial v(0,\ t)}{\partial x} = \frac{\lambda_r}{b}[v(0,\ t) - u(0,\ t)] - \frac{Q}{2}\delta(t),$$
$$\lambda_0 \frac{\partial w(d,\ t)}{\partial x} = \frac{\lambda_r}{b}[w(d,\ t) - v(d,\ t)], \qquad (3)$$
$$\lambda \frac{\partial v(d,\ t)}{\partial x} = \frac{\lambda_r}{b}[w(d,\ t) - v(d,\ t)]$$

and the condition of boundedness of the temperature at infinity

$$u(-\infty,\ t) = 0,\quad w(\infty,\ t) = 0. \qquad (4)$$

The conditions of coupling between functions u(0, t) and v(0, t) in expressions (3) are obtained from the solution to the heat-conduction equation for the layer ($0 < x < b$), assuming that its thermal conductivity is zero and all the heater energy is released instantaneously at the section $x = b/2$ only, i.e., at the center of the layer. Analogously, from the solution to the heat-conduction equation for the layer ($d < x < d + b$) are obtained the conditions of coupling between functions v(d, t) and w(d, t).

Applying the Laplace transformation to system (1) with conditions (2) and (3) yields

$$U'' - \frac{p}{a_0}U = 0,$$
$$V'' - \frac{p}{a}V = 0, \qquad (5)$$
$$W'' - \frac{p}{a_0}W = 0,$$

where U, V, and W are the respective transforms of the functions.

The boundary conditions become

$$\lambda_0 U'(0) = \frac{\lambda_r}{b}[V(0) - U(0)] + Q/2,$$
$$\lambda V'(0) = \frac{\lambda_r}{b}[V(0) - U(0)] - Q/2,$$
$$\lambda V'(d) = \frac{\lambda_r}{b}[W(d) - V(d)], \qquad (6)$$
$$\lambda_0 W'(d) = \frac{\lambda_r}{b}[W(d) - V(d)].$$

The general solution to system (5) can be written as

$$U(x,\ p) = A\exp(-\sqrt{p/a_0}x) + B\exp(\sqrt{p/a_0}x),$$
$$V(x,\ p) = C\exp(-\sqrt{p/a}x) + D\exp(\sqrt{p/a}x), \qquad (7)$$
$$W(x,\ p) = G\exp(-\sqrt{p/a_0}x) + H\exp(\sqrt{p/a_0}x).$$

Conditions (4) yield A = 0 and H = 0. From solution (7) with conditions (6) we determine the constants B, C, D, and G:

$$B = \left\{\frac{Q(n+2k)}{2[mn+(m+n)k} + \frac{Q(n-2k)[(m-n)k-mn]}{2\alpha^2[mn+(m+n)k]^2}\right\}\sum_{r=0}^{\infty}\frac{[(m-n)k-mn]^{2r}}{\alpha^{2r}[mn+(m+n)k]^{2r}}, \qquad (8)$$

$$C = \frac{Q(m+2k)}{2[mn+(m+n)k]}\sum_{r=0}^{\infty}\frac{[(m-n)k-mn]^{2r}}{\alpha^{2r}[mn+(m+n)k]^{2r}}, \qquad (9)$$

$$D = \frac{Q(m+2k)[mn-(m-n)k]}{2\alpha^2[mn+(m+n)k]^2}\sum_{r=0}^{\infty}\frac{[(m-n)k-mn]^{2r}}{\alpha^{2r}[mn+(m+n)k]^{2r}}, \qquad (10)$$

$$G = \frac{Q\beta(m+2k)kn}{\alpha[mn+(m+n)k]^2}\sum_{r=0}^{\infty}\frac{[(m-n)k-mn]^{2r}}{\alpha^{2r}[mn+(m+n)k]^{2r}}. \qquad (11)$$

Here $k = \lambda_r/b$ [W/m·C] is the heat conductance of the thermal contact region, $m = \lambda_0\sqrt{p/a_0}$; $n = \sqrt{p/a}$; $\alpha = \exp(\sqrt{p/a}d)$; $\beta = \exp(\sqrt{p/a_0}d)$.

For high values of p, corresponding to the time period of interest here, series (8)-(11) converge so fast that already the first term approximates the sum with sufficient accuracy. Accordingly, only one term of the sums representing the constants B, C, D, and G will be retained for insertion into the general solution (7).

For determining the thermophysical properties it is necessary to obtain an expression for the functions of the temperature within the layer containing the thermometer v(d, t) and w(d + b, t).

We calculate the function V(x, p) by inserting the values of constants C and D into the second equality in the solution (7). After several transformations we obtain

$$V(x,\ p) = \frac{Q}{2\eta}\frac{\exp(-\gamma\sqrt{p})}{(\sqrt{p}+sk)} + \frac{Qk}{\eta\eta_0}\frac{\exp(-\gamma\sqrt{p})}{\sqrt{p}(\sqrt{p}+sk)} +$$
$$+\frac{Q}{2\eta}\frac{\sqrt{p}\exp(-\nu\sqrt{p})}{(\sqrt{p}+sk)^2} + \frac{Qk(3\eta-\eta_0)}{2\eta_0\eta^2}\frac{\exp(-\nu\sqrt{p})}{(\sqrt{p}+sk)^2} - \frac{Qk^2g}{\eta_0^2\eta^2}\frac{\exp(-\nu\sqrt{p})}{\sqrt{p}(\sqrt{p}+sk)^2}, \qquad (12)$$

where $\eta = \lambda/\sqrt{a}$; $\eta_0 = \lambda_0/\sqrt{a_0}$; $\gamma = x/\sqrt{a}$; $\nu = (2d - x)/\sqrt{a}$; $f = \eta_0 + \eta$; $g = \eta_0 - \eta$; and $s = f/\eta\eta_0$. An inverse Laplace transformation of expression (12) yields [8]

$$v(x,\ t) = \frac{Q}{2\eta\sqrt{\pi t}}\exp(-\gamma^2/4t) - \frac{Qkg}{2\eta^2\eta_0}\exp(\gamma sk + s^2k^2t)\times$$
$$\times\operatorname{erfc}[(\gamma/2\sqrt{t}) + sk\sqrt{t}] + Q\left[\frac{1+2s^2k^2t}{2\eta\sqrt{\pi t}} - \frac{sk^2(3\eta-\eta_0)\sqrt{t}}{\eta_0\eta^2\sqrt{\pi}} - \right.$$
$$\left. - \frac{2gk^2\sqrt{t}}{\eta_0^2\eta\sqrt{\pi}}\right]\exp(-\nu^2/4t) + Q\left[-\frac{sk(2+2s^2k^2t+\nu sk)}{2\eta} + \right.$$
$$\left. + \frac{k(3\eta-\eta_0)(1+sk\nu+2s^2k^2t)}{2\eta_0\eta^2} + \frac{k^2g(2skt+\nu)}{\eta_0^2\eta^2}\right]\times$$
$$\times\exp(\nu sk + s^2k^2t)\operatorname{erfc}[(\nu/2\sqrt{t}) + sk\sqrt{t}].$$

We now expand function erfc(z) asymptotically for large values of the argument [9]. Retaining the first two terms of the expansion

$$v(x,\ t)=Q\left\{\frac{1}{2\eta\sqrt{\pi t}}-\frac{gk\sqrt{t}}{\eta^2\eta_0\sqrt{\pi}(\gamma+2skt)}+\frac{2gkt\sqrt{t}}{\eta^2\eta_0\sqrt{\pi}(\gamma+2skt)^3}\right\}\exp(-\gamma^2/4t)+$$

$$+Q\left\{\frac{1}{2\eta\sqrt{\pi t}}+\frac{k(\eta-3\eta_0)\sqrt{t}}{\eta^2\eta_0\sqrt{\pi}(\nu+2skt)}+\frac{4gk^2t\sqrt{t}}{\eta^3\eta_0\sqrt{\pi}(\nu+2skt)^2}-\frac{2k(\eta-3\eta_0)t\sqrt{t}}{\eta^2\eta_0\sqrt{\pi}(\nu+2skt)^3}\right\}\exp(-\nu^2/4t) \quad (13)$$

will ensure the required accuracy. Inserting the value of constant G into the third equality in the general solution (7) will yield the function W(x, p):

$$W(x,\ p)=\frac{Q\beta(m+2k)kn}{\alpha[mn+(m-n)k]^2}\exp(-\sqrt{p/a_0}x)$$

or, in the adopted notation,

$$W(x,\ p)=\frac{Qk}{\eta\eta_0}\frac{\exp(-\mu\sqrt{p})}{(\sqrt{p}+sk)^2}+\frac{2Qk^2}{\eta\eta_0^2}\frac{\exp(-\mu\sqrt{p})}{\sqrt{p}(\sqrt{p}+sk)^2}. \quad (14)$$

Here $\mu=(d/\sqrt{a})+(x-d)/\sqrt{a_0}$. An inverse Laplace of expression (14) yields

$$w(x,\ t)=-\frac{2Qk^2g\sqrt{t}}{\eta^2\eta_0^2\sqrt{\pi}}\exp(-\mu^2/4t)+Q\left[\frac{k}{\eta\eta_0}+\frac{gk^2(2skt+\mu)}{\eta^2\eta_0}\right]\exp(sk\mu+s^2k^2t)\operatorname{erfc}[(\mu/2\sqrt{t})+sk\sqrt{t}].$$

Retaining the first two terms in the asymptotic expansion of the supplementary error function yields, after a few transformations,

$$w(x,\ t)=\frac{2Qk\sqrt{t}}{\eta\eta_0\sqrt{\pi}(\mu+2skt)}\left[1-\frac{2gkt}{\eta\eta_0(\mu+2skt)}-\frac{2t}{(\mu+2skt)^2}\right]\exp(-\mu^2/4t). \quad (15)$$

The thermophysical characteristics of the tested material and the thermal conductivity of the contact region are found as follows. We select n points on the experimental curve and denote the temperatures at these points as θ_i, i = 1, 2, 3, ..., n. We then examine the function

$$F(a,\ \lambda,\ k)=\max_{1\leqslant i\leqslant n}\left(\frac{w_i-\theta_i}{\theta_i}\right)^2, \quad (16)$$

where w_i is the temperature calculated according to expression (15) for the i-th point.

The problem of determining the unknown values of a, λ, and k reduces to minimizing the function F(a, λ, k), its minimum corresponding to triad of values a, λ, k closest to the true ones.

The zeroth-order approximation for a and λ is obtained from the solution to the heat-conduction problem (1) for an ideal thermal contact between the media within the region of their interface [10]. The zeroth-order approximation for k is obtained from the condition of contact between the tested specimen and the reference material, with the thermal conductivity of the heater material and the thermometer material as well as the thickness of both taken into account.

Some test data on the thermal conductivity and the thermal diffusivity of specimens certified at the D. I. Mendeleev All-Union Scientific-Research Institute of Metrology are given in Table 1, based on the initial approximation k = 800 W/m·C. The errors of λ and a determinations, their absolute values, do not exceed 5-6%. The data in this table indicate that the error of a decreases with increasing thickness of the specimen. This can be explained by the decreasing effect of thermal resistance in the contact. The error of λ increases with increasing thickness of the specimen. This is apparently due to heat dissipation through the lateral surface of a specimen, which begins to become significant as the dimension d of the specimen increases. The spread of k values based on those tests is attributable to different conditions at the contact, i.e., different pressures in the contact region, different degrees of surface roughness, etc.

When the ratio of thermal resistance of the specimen to thermal resistance in the contacts is relatively large (larger than 10), then it is permissible to use the asymptotic expansion of expression (15) in powers of k_0/k ($k_0 = \lambda/d$ denoting the thermal conductance of the specimen).

TABLE 1. Experimental Data on the Thermophysical Properties of Fused Quartz and Acrylic Glass

Item No.	Specimen	d, mm	λ_0, W/m·deg	$a_0 \cdot 10^6$, m²/sec	$F_{min} \cdot 10^6$	k, W/m·deg	λ, W/m·deg	$a \cdot 10^6$, m²/sec	$\delta\lambda$, %	δa, %
1	Acrylic glass	5,05	0,197	0,113	2,24	970	0,1125	0,207	5,1	—0,5
2		3,63	0,197	0,113	0,65	550	0,116	0,203	3,0	—2,6
3	Fused quartz	4,00	0,197	0,113	3,11	970	0,794	1,31	—2,1	—4,4
4		3,50	0,197	0,113	50,1	1140	0,781	1,33	—0,6	—5,9
5		4,99	1,34	0,83	31,5	1066	0,789	1,43	6,0	—4,9

We will now expand the function W(x, p) in expression (14) into a power series with respect to 1/k:

$$W(x,\ p) \sim \frac{Q\exp(-\mu\sqrt{p})}{f^2\sqrt{p}}\left[\sum_{m=2}^{\infty}(-1)^m m\eta(\sqrt{p}/sk)^{m-2} - \sum_{m=1}^{\infty}(-1)^m m\eta_0(\sqrt{p}/sk)^m\right].$$

This expression will be rewritten as

$$W(x,\ p) \sim \frac{2Q\eta\exp(-\mu\sqrt{p})}{f^2\sqrt{p}} - \frac{Q(\eta_0 - 3\eta)\exp(-\mu\sqrt{p})}{f^2 sk} +$$

$$+\frac{Q}{f^2}\left\{\sum_{m=1}^{\infty} 2\left[m(\eta - \eta_0) + \eta\right](\sqrt{p}/sk)^{2m}\exp(-\mu\sqrt{p}) -\right.$$

$$\left. - \sum_{m=1}^{\infty}\left[(2m+1)(\eta - \eta_0) + 2\eta\right](\sqrt{p}/sk)^{2m+1}\exp(-\mu\sqrt{p})\right\}.$$

Applying the correspondence theorem to operations on originals and on transforms during differentiation of the original, and then equating the derivative $\partial^m w(x, t)/\partial t^m$ (m = 1, 2, 3, ...) at t = +0 to zero, we obtain

$$w(x,\ t) \sim \frac{Q\exp(-\mu^2/4t)}{f^2\sqrt{\pi t}}\left[2\eta - \frac{\mu(\eta_0 - 3\eta)}{2skt}\right] + \frac{Q}{f^2}\left\{\sum_{m=1}^{\infty}\frac{2\left[m(\eta - \eta_0) + \eta\right]}{(sk)^{2m}}\frac{d^m}{dt^m}\left[\frac{\exp(-\mu^2/4t)}{\sqrt{\pi t}}\right] -\right.$$

$$\left. - \sum_{m=1}^{\infty}\frac{\left[(2m+1)(\eta - \eta_0) + 2\eta\right]}{(sk)^{2m+1}}\frac{d^m}{dt^m}\left[\frac{\mu\exp(-\mu^2/4t)}{2t\sqrt{\pi t}}\right]\right\}.$$

We next introduce the dimensionless variables Fo = at/d^2 and Bi = k/k_0 so that, after a few transformations, we obtain for x = d

$$w(\text{Fo},\ d) \sim \frac{Q\exp(-1/4\text{Fo})\sqrt{a}}{f^2 d\sqrt{\pi\,\text{Fo}}}\left[2\eta - \frac{(3\eta - \eta_0)\eta_0}{2f\,\text{Fo}\,\text{Bi}} +\right.$$

$$\left. + \frac{(4\eta - 2\eta_0)(1 - 2\text{Fo})\eta_0^2}{4f^2\text{Fo}^2\text{Bi}^2} - \frac{(5\eta - 3\eta_0)(1 - 6\text{Fo})\eta_0^3}{8f^3\text{Fo}^3\text{Bi}^3} + \ldots\right].$$

We will now describe the computer procedure for seeking the minimum of function F(a, λ, k). The program for determining the quantities a, λ, and k has been written in the GDR-ALGOL language for a model BÉSM-6 high-speed computer. It includes the standard routines MINIG [11] and DIRECT [12] for seeking the minimum, combined with the method of a trough step.

We note that the relief of function F(a, λ, k) has in many cases a rather intricate pattern with steep descents, troughs, and several local minima. Some experience is, therefore, necessary for operating the program and selecting the trough step.

As an example, in Figs. 1-3 are shown isolines of function F(a, λ, k) segments near its minimum. These graphs represent the results of test No. 2 (Table 1).

In conclusion, we will note that in this study a series of tests was performed with the thermal conductance k of the contact and k_0 of the specimen comparable in magnitude (i.e., with small values of the Biot number). Therefore, this method can be recommended for determining the thermophysical properties of high-conductivity materials and allows the use of thin specimens.

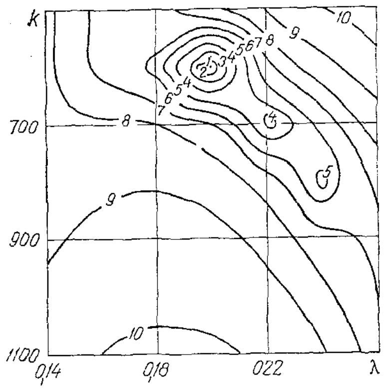

Fig. 1

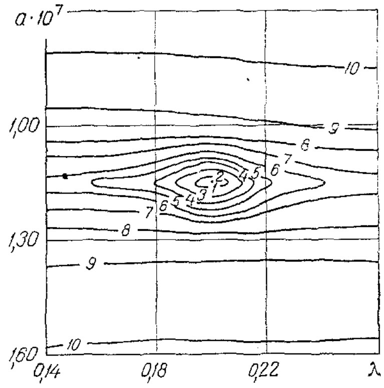

Fig. 2

Fig. 1. Behavior of function F(a, λ, k) near its minimum (a = 1.15·10^{-7} m^2/sec); values of function φ = −log F: 1) 5.90; 2) 5.53; 3) 5.15; 4) 4.78; 5) 4.40; 6) 4.03; 7) 3.65; 8) 3.28; 9) 2.90; 10) 2.53. k (W/m^2·C), λ(W/m·C).

Fig. 2. Behavior of function F(a, λ, k) near its minimum (k = 600 W/m^2·C); values of function φ = −log F: 1) 5.88; 2) 5.31; 3) 4.74; 4) 4.17; 5) 3.60; 6) 3.04; 7) 2.47; 8) 1.90; 9) 1.33; 10) 0.76. a·10^7 m^2/sec.

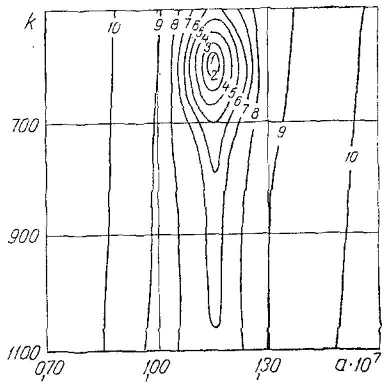

Fig. 3. Behavior of function F(a, λ, k) near its minimum (λ = 0.2 W/m·C); values of function φ = −log F: 1) 5.88; 2) 5.31; 3) 4.75; 4) 4.18; 5) 3.61; 6) 3.05; 7) 2.48; 8) 1.92; 9) 1.35; 10) 0.79.

APPENDIX: DESCRIPTION OF THE PROGRAM FOR DETERMINING THE QUANTITIES a, λ, k

Identifiers: M, number of points on the experimental curve; KP, number of variables of the function to be minimized; MIZ, value of the function to be minimized; TM, time to reach the maximum temperature; TETAM, maximum temperature on the experimental curve;

$$ET0 = \eta_0 \quad A0 = a_0 \quad L0 = \lambda_0 \quad TETA[I] = \theta_i$$
$$PI = \pi \quad B = d \quad T[I] = t_i \quad Q = Q$$

KU, AU, LU are the initial approximations for k, a, and λ, respectively; WE, procedure-function which realizes expression (15); S2, procedure-function for the function (16) to be minimized; MINIG, standard procedure for minimum search by the method of steepest descent; MINIF, auxiliary procedure-function for implementing the MINIG procedure; OWRAG, procedure for minimum search in the case of a trough relief of function F(a, λ, k), implementing the method of the trough step; DIRECT, standard procedure of minimum search by the method of steepest descent along coordinates; and EXSTEP, standard procedure for reverting the control from computing to deck processing.

The program operates as follows. After a read-in of the input data, the variables Q, AO, B, and T are normalized so as to avoid an overflow of the arithmetic unit in the computer. Then from the TETA variables array is selected the largest element, and the initial approximations for λ and a are found by the method described elsewhere [10]. The initial approximation for k is stipulated. The zeroth-order approximations come printed out.

The first approximation for $F(a, \lambda, k)$ is found by the MINIG procedure. The function $F(a, \lambda, k)$ is minimized by the OWRAG procedure. A more precise minimization near the global minimum is performed according to the DIRECT procedure. The values of the minimized function MIZ, the sought values of a, λ, k, the instants of time t_i, the values of function $w_i(k, a, \lambda)$ at time t_i, calculated according to expression (15), and the corresponding temperatures θ_i at these points are all printed out.

NOTATION

a and a_0, thermal diffusivity of the tested material and the reference material, respectively; d, thickness of the test specimen; k and k , thermal conductance in the contact and of the specimen, respectively; t, current time; u, v, w, functions of the temperature; η and η_0, thermal activity coefficients of the tested material and the reference material, respectively; λ, λ_0, λ_r, thermal conductivity of the tested material, the reference material, and in the contact region, respectively; F_{min}, minimum value of function $F(a, \lambda, k)$; x, coordinate; $\delta\lambda$, relative error of a λ determination; and δa, relative error of an a determination.

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